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$h_{\mu\lambda}(k)$, $k \in \mathrm{T}^3$ qatlam operatorini simmetrik va antisimmetrik qism fazolar yodamida fredgolm determenantini topish

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Annotatsiya: Ushbu ishda uch o'lchamli panjarada $h_{\mu\lambda}(k)$, $k \in \mathrm{T}^3$ qatlam operatorini simmetrik va antisimmetrik qism fazolar yodamida fredgolm determenantini topilgan. Ushbu Fredgolm detirminanti berilgan fazoni ikkita invariant qism fazolarga yoyish orqali oson usulda topilgan.

Kalit so‘zlar: Fredgolm detirminanti, operator, xos qiymat, spektr, muhim spektr.

Kirish: Tashqi maydon ta'siridagi bir kvant zarracha harakatini tavsiflovchi bir zarrachali gamiltonianlar hamda qisqa masofada ta'sirlashuvchi ikkita bir xil va har xil zarrachalar sistemasi gamiltonianlariga mos ikki zarrachali diskret Shredinger operatorlarining spektral xossalari Faria, Corolli, S.Albeverio, S.Laqayevlarning ishlarda o'rganilgan.

Ikki zarrachali diskret Shredinger operatori $h_\mu(0)$ bir zarrachali gamiltoniani h_μ ga unitar ekvivalent va shuning uchun bir zarrachali Shredinger operatorlarining spektral xossalarini o'rganish panjaradagi ko'p zarrachali Shredinger operatorlar spektral nazariyasida muhim o'rin tutadi.

$h_{\mu\lambda}(k)$, $k \in \mathrm{T}^3$ qatlam operatori $L^{2,e}(T^3)$ fazoda o‘z-o‘ziga qo‘shma operator quyidagi formula bilan aniqlanadi

$$h_{\mu\lambda}(k) = h_0(k) + v_{\mu\lambda}, \tag{1}$$

bu yerda $h_0(k)-$ $\varepsilon_k(\cdot)$ funksiyaga ko'paytirish operatori:

$$\varepsilon_k(p) = 2\sum_{i=1}^{3}\left(1-\cos\frac{k_i}{2}\cos p_i\right) \qquad (2)$$

va $v_{\mu\lambda}$ – integral operator va uning rangi 4 dan oshmaydi:

Faraz qilaylik $k=0$ bo'lsin. Ushbu $L^{2,e,}(T^3)$ juft funksiyalar fazosining *simmetrik va antisimmetrik* qism fazolari

$$L^{2,e,s}(T^3):=\left\{f\epsilon L^{2,e}\left(\mathrm{T}^3\right): f(p_1,p_2,p_3) = \prod_2 f(p_1, p_2 p_3),\ \ p_1,p_2,p_3\epsilon\mathrm{T}\right\}$$

va

$$L^{2,e,a}(T^3):=\left\{f\epsilon L^{2,e}\left(\mathrm{T}^3\right): f(p_1,p_2,p_3) = -\prod_2 f(p_1, p_2 p_3),\ \ p_1,p_2,p_3\epsilon\mathrm{T}\right\},$$

$h_{\mu\lambda}(0)$ operator ta'siriga nisbatan invariant qism fazolar bo'ladi, bu yerda Π_2 ixtiyoriy ikkita p_i va p_j *i*, *j*=1,2,3 o'zgaruvchilarning o'rin almashtirishi.

***$h_{\mu\lambda}(0)$* operatorning diskret spektri.** Bizga ma'lumki, $L^{2,e,s}(\mathrm{T}^3)$ va $L^{2,e,a}(\mathrm{T}^3)$ Hilbert fazolari $h_{\mu\lambda}(0)$ operatorga nisbatan invariant. $h_{\mu\lambda}(0)$ operatorning $L^{2,e,s}(T^3)$ va $L^{2,e,a}(T^3)$ fazolardagi qismi $h_{\mu\lambda}(0)_{L^{2,e,s}(T^3)}$ va $h_{\mu\lambda}(0)_{L^{2,e,a}(T^3)}$ ni mos ravishda $h_{\mu\lambda}^{(s)}$ va h_{λ}^{a} orqali belgilaymiz. $h_{\mu\lambda}^{(s)}$ va h_{λ}^{a} operatorlar $L^{2,e,s}(T^3)$ va $L^{2,e,a}(T^3)$ da quyidagi formulalar bilan aniqlanadi

$$h_{\mu\lambda}^{(s)} := h_0(0) + v_{\mu\lambda}^{s} \text{ va } h_{\lambda}^{a} := h_0(0) + v_{\lambda}^{a},$$

bu yerda $v_{\mu\lambda}^{(s)}$ va v_{λ}^{a} integral operatorlar quyidagi formula bilan berilgan.

$$v_{\mu\lambda}^{s} f(p) = \frac{\mu}{(2\pi)^3}\int_{T^3} f(q)dq + \frac{\lambda}{(2\pi)^3}\frac{1}{4}\sum_{i=1}^{2}\sum_{j=i+1}^{3}(cosp_i + cosp_j)\int_{T^3}(cosq_i - cosq_j)f(q)dq$$

va

$$v_{\lambda}^{a}f(p)=\frac{\lambda}{(2\pi)^3}\frac{1}{4}\sum_{i=1}^{2}\sum_{j=i+1}^{3}(cosp_i-cosp_j)\int_{T^3}(cosq_i-cosq_j)f(q)dq$$

$h_0(0)$ operator $\varepsilon_0(\cdot)=2\varepsilon(\cdot)\epsilon L^{2,e}(T^3)$ simmetrik funksiya bo'yicha ko'paytirish operatori. Shuning uchun ikkala $L^{2,e,s}(T^3)$ va $L^{2,e,a}(T^3)$ qism fazolar $h_0(0)$ operatorga invariant.

Lemma 2.2. $\boldsymbol{h_{\mu\lambda}^{(s)}(0)}$ va $\boldsymbol{h_{\lambda}^{(a)}(0)}$ operatorlarga mos Fredgolm determinantlari quyidagi ko'rinishga ega bo'ladi:

$\Delta_{\mu\lambda}^{(s)}(z):=\Delta_{\mu0}(z)\Delta_{0\lambda}(z)-3\mu\lambda b^2(z),$

va

$$\Delta_{\lambda}^{(a)}(z):=\left[1+\lambda\big(c(z)-e(z)\big)\right]^2.$$

Isbot.

$h_{\mu\lambda}^{(s)}(0)=h_0(0)+v_{\mu\lambda}^{(s)},\ (h_0(0)f)(p)=\varepsilon_0(p)f(p),\quad f\epsilon L^{2,e}(T^3)$

$$\varepsilon_0(p)=\sum_{i=1}^{3}(1-cosp_i),\qquad p=(p_1,p_2,p_3)\epsilon T^3,$$

$$v_{\mu\lambda}^{(s)}f(p)=\frac{\mu}{(2\pi)^3}\int_{T^3}f(q)dq+\frac{\lambda}{(2\pi)^3}\frac{1}{4}\sum_{i=1}^{2}\sum_{j=i+1}^{3}(cosp_i$$

$$+cosp_j)\int_{T^3}(cosq_i-cosq_j)f(q)dq$$

$h_{\mu\lambda}^{(s)}f=zf,\ z\epsilon\backslash[0,12],\ f\neq 0$ tenglamani yechamiz.

$$(h_0-v_{\mu\lambda}^{(s)})f(p)=zf(p)$$

$$h_0f(p)-v_{\mu\lambda}^{(s)}f(p)=zf(p)$$

$$\varepsilon_0(p)f(p)-zf(p)=v_{\mu\lambda}^{(s)}f(p)$$

$f(p)=\frac{v_{\mu\lambda}^{(s)}f(p)}{\varepsilon_0(p)-z},$

$$f(p)=\frac{\mu}{(2\pi)^3}\int_{T^3}\frac{f(q)dq}{\varepsilon_0(p)-z}+\frac{\lambda}{32\pi^3}(cosp_1+cosp_2)$$
$$\times\int_{T^3}\frac{(cosq_1+cosq_2)f(q)dq}{\varepsilon_0(p)-z}+\frac{\lambda}{32\pi^3}(cosp_1+cosp_3)$$
$$\times\int_{T^3}\frac{(cosq_1+cosq_3)f(q)dq}{\varepsilon_0(p)-z}+\frac{\lambda}{32\pi^3}(cosp_2+cosp_3)$$
$$\times\int_{T^3}\frac{(cosq_2+cosq_3)f(q)dq}{\varepsilon_0(p)-z}=$$
$$=\frac{\mu}{(2\pi)^3}\int_{T^3}\frac{f(q)dq}{\varepsilon_0(p)-z}+\frac{\lambda cosp_1}{32\pi^3}\left(\int_{T^3}\frac{cosq_1f(q)dq}{\varepsilon_0(p)-z}+\int_{T^3}\frac{cosq_2f(q)dq}{\varepsilon_0(p)-z}\right)+$$
$$+\frac{\lambda cosp_2}{32\pi^3}\left(\int_{T^3}\frac{cosq_1f(q)dq}{\varepsilon_0(p)-z}+\int_{T^3}\frac{cosq_2f(q)dq}{\varepsilon_0(p)-z}\right)+$$
$$+\frac{\lambda cosp_1}{32\pi^3}\left(\int_{T^3}\frac{cosq_1f(q)dq}{\varepsilon_0(p)-z}+\int_{T^3}\frac{cosq_3f(q)dq}{\varepsilon_0(p)-z}\right)+$$
$$\frac{\lambda cosp_3}{32\pi^3}\left(\int_{T^3}\frac{cosq_1f(q)dq}{\varepsilon_0(p)-z}+\int_{T^3}\frac{cosq_3f(q)dq}{\varepsilon_0(p)-z}\right)+$$
$$+\frac{\lambda cosp_2}{32\pi^3}\left(\int_{T^3}\frac{cosq_2f(q)dq}{\varepsilon_0(p)-z}+\int_{T^3}\frac{cosq_3f(q)dq}{\varepsilon_0(p)-z}\right)+$$
$$+\frac{\lambda cosp_3}{32\pi^3}\left(\int_{T^3}\frac{cosq_2f(q)dq}{\varepsilon_0(p)-z}+\int_{T^3}\frac{cosq_3f(q)dq}{\varepsilon_0(p)-z}\right)$$

Quyidagi belgilashlarni kiritamiz:

$C_0=\frac{1}{(2\pi)^3}\int_{T^3}\frac{f(q)dq}{\varepsilon_0(p)-z}$, $C_i=\frac{1}{32\pi^3}\int_{T^3}\frac{cosq_if(q)dq}{\varepsilon_0(p)-z}$, i=1,2,3 $C_1=C_2=C_3$

$$f(p)=\mu C_0+4\lambda C_1cosp_1+4\lambda C_1cosp_2+4\lambda C_1cosp_3$$
$$f(p)=\mu C_0+4\lambda C_1(cosp_1+cosp_2+cosp_3)$$
$$f(q)=\mu C_0+4\lambda C_1(cosq_1+cosq_2+cosq_3)$$

$$\begin{cases} C_0 = \dfrac{1}{(2\pi)^3} \displaystyle\int_{T^3} \dfrac{(\mu C_0 + 4\lambda C_1 (cosq_1 + cosq_2 + cosq_3) dq}{\varepsilon_0(q) - z} \\ C_1 = \dfrac{1}{32\pi^3} \displaystyle\int_{T^3} \dfrac{cosq_1(\mu C_0 + 4\lambda C_1 (cosq_1 + cosq_2 + cosq_3) dq}{\varepsilon_0(q) - z} \end{cases}$$

$$\begin{cases} C_0 = \dfrac{1}{(2\pi)^3} \displaystyle\int_{T^3} \dfrac{\mu C_0 dq}{\varepsilon_0(q) - z} + \dfrac{4\lambda C_1}{(2\pi)^3} \displaystyle\int_{T^3} \dfrac{cosq_1 dq}{\varepsilon_0(q) - z} + \\ + \dfrac{4\lambda C_1}{(2\pi)^3} \displaystyle\int_{T^3} \dfrac{cosq_2 dq}{\varepsilon_0(q) - z} + \dfrac{4\lambda C_1}{(2\pi)^3} \displaystyle\int_{T^3} \dfrac{cosq_3 dq}{\varepsilon_0(q) - z} \\ C_1 = \dfrac{\mu C_0}{32\pi^3} \displaystyle\int_{T^3} \dfrac{cosq_1 dq}{\varepsilon_0(q) - z} + \dfrac{\lambda C_1}{(2\pi)^3} \displaystyle\int_{T^3} \dfrac{cos^2 q_1 dq}{\varepsilon_0(q) - z} + \\ + \dfrac{\lambda C_1}{(2\pi)^3} \displaystyle\int_{T^3} \dfrac{cosq_1 cosq_2 dq}{\varepsilon_0(q) - z} + \dfrac{\lambda C_1}{(2\pi)^3} \displaystyle\int_{T^3} \dfrac{cosq_1 cosq_3 dq}{\varepsilon_0(q) - z} \end{cases}$$

$a(z) := \frac{1}{(2\pi)^3} \int_{T^3} \frac{dq}{\varepsilon_0(q) - z}$, $b(z) := \frac{1}{(2\pi)^3} \int_{T^3} \frac{cosq_i dq}{\varepsilon_0(q) - z}$,

$c(z) := \frac{1}{(2\pi)^3} \int_{T^3} \frac{cos^2 q_i dq}{\varepsilon_0(q) - z}$, $e(z) := \frac{1}{(2\pi)^3} \int_{T^3} \frac{cosq_i cosq_j dq}{\varepsilon_0(q) - z}$, $i, j = 1,2,3\ i \neq j$

$$\begin{cases} C_0 = \mu C_0 a(z) + 12\lambda C_1 b(z) \\ C_1 = \dfrac{\mu C_0}{4} b(z) + \lambda C_1 c(z) + 2\lambda C_1 e(z) \end{cases}$$

$$\begin{cases} C_0(1 - \mu a(z) - 12 C_1 \lambda b(z) = 0 \\ C_0 \dfrac{\mu}{4} b(z) + C_1(\lambda c(z) + 2\lambda e(z) - 1) = 0 \end{cases}$$

bo'lganda Fredgolm determinantini tuzamiz.

$$\Delta_{\mu\lambda}^{(s)}(z) = \begin{vmatrix} 1 - \mu a(z) & -12\lambda b(z) \\ \dfrac{\mu b(z)}{4} & (\lambda c(z) + 2\lambda e(z) - 1) \end{vmatrix} = 0$$

$$\Delta_{\mu\lambda}^{(s)}(z) = \big(1 + \mu a(z)\big)(\lambda c(z) + 2\lambda e(z) + 1) - 3\mu\lambda b^2(z)$$

Endi antisimmetrik holat uchun ko'rsatamiz:

$h_\lambda^{(a)}(0) = h_0(0) + v_\lambda^{(a)}$, $(h_0(0)f)(p) = \varepsilon_0(p)f(p)$, $f \epsilon L^{2,e}(T^3)$

$$\varepsilon_0(p) = \sum_{i=1}^{3}(1 - cosp_i), \qquad p = (p_1, p_2, p_3)\epsilon T^3,$$

$$v_\lambda^a f(p) = \frac{\lambda}{(2\pi)^3}\frac{1}{4}\sum_{i=1}^{2}\sum_{j=i+1}^{3}(cosp_i - cosp_j)\int_{T^3}(cosq_i - cosq_j)f(q)dq$$

$h_\lambda^{(a)}f = zf$, $z\epsilon\backslash[0,12]$, $f \neq 0$ tenglamani yechamiz.

$$(h_0 - v_\lambda^{(a)})f(p) = zf(p)$$

$$h_0 f(p) - v_\lambda^{(a)} f(p) = zf(p)$$

$$\varepsilon_0(p)f(p) - zf(p) = v_\lambda^{(a)} f(p)$$

$f(p) = \frac{v_\lambda^{(a)} f(p)}{\varepsilon_0(p) - z}$,

$$f(p) = \frac{\lambda}{32\pi^3}\sum_{i=1}^{2}\sum_{j=i+1}^{3}(cosp_i - cosp_j)\int_{T^3}\frac{(cosq_i - cosq_j)f(q)dq}{\varepsilon_0(p) - z}$$

$$f(p) = \frac{\lambda}{32\pi^3}(cosp_1 - cosp_2)\int_{T^3}\frac{(cosq_1 - cosq_2)f(q)dq}{\varepsilon_0(p) - z}$$

$$+ \frac{\lambda}{32\pi^3}(cosp_1 - cosp_3)\int_{T^3}\frac{(cosq_1 - cosq_3)f(q)dq}{\varepsilon_0(p) - z} + \frac{\lambda}{32\pi^3}(cosp_2$$

$$- cosp_3)\int_{T^3}\frac{(cosq_2 - cosq_3)f(q)dq}{\varepsilon_0(p) - z}$$

$$f(p)=\frac{\lambda cosp_1}{32\pi^3}(\int_{T^3}\frac{cosq_1 f(q)dq}{\varepsilon_0(p)-z}-\int_{T^3}\frac{cosq_2 f(q)dq}{\varepsilon_0(p)-z})$$

$$-\frac{\lambda cosp_2}{32\pi^3}(\int_{T^3}\frac{cosq_1 f(q)dq}{\varepsilon_0(p)-z}-\int_{T^3}\frac{cosq_2 f(q)dq}{\varepsilon_0(p)-z})$$

$$+\frac{\lambda cosp_1}{32\pi^3}(\int_{T^3}\frac{cosq_1 f(q)dq}{\varepsilon_0(p)-z}-\int_{T^3}\frac{cosq_3 f(q)dq}{\varepsilon_0(p)-z})$$

$$-\frac{\lambda cosp_3}{32\pi^3}(\int_{T^3}\frac{cosq_1 f(q)dq}{\varepsilon_0(p)-z}-\int_{T^3}\frac{cosq_3 f(q)dq}{\varepsilon_0(p)-z})$$

$$+\frac{\lambda cosp_2}{32\pi^3}(\int_{T^3}\frac{cosq_2 f(q)dq}{\varepsilon_0(p)-z}-\int_{T^3}\frac{cosq_3 f(q)dq}{\varepsilon_0(p)-z})$$

$$-\frac{\lambda cosp_3}{32\pi^3}(\int_{T^3}\frac{cosq_2 f(q)dq}{\varepsilon_0(p)-z}-\int_{T^3}\frac{cosq_3 f(q)dq}{\varepsilon_0(p)-z})$$

Quyidagi belgilashlarni kiritamiz:

$C_i=\frac{1}{(2\pi)^3}\int_{T^3}\frac{cosq_i f(q)dq}{\varepsilon_0(p)-z}$, i=1,2,3 $C_1=C_2=C_3$

$$f(p)=C_1\lambda(cosp_1-cosp_3)\qquad f(q)=C_1\lambda(cosq_1-cosq_3)$$

$$C_1=\frac{1}{(2\pi)^3}\int_{T^3}\frac{cosq_1 C_1\lambda(cosq_1-cosq_3)dq}{\varepsilon_0(q)-z}$$

$$C_1=\frac{C_1\lambda}{(2\pi)^3}(\int_{T^3}\frac{cos^2q_1dq}{\varepsilon_0(q)-z}-\int_{T^3}\frac{cosq_1cosq_3dq}{\varepsilon_0(q)-z}$$

Quyidagi belgilashlardan foydalanamiz:

$c(z)=\frac{1}{(2\pi)^3}(\int_{T^3}\frac{cos^2q_1dq}{\varepsilon_0(q)-z}$ $\qquad$ $e(z)=\frac{1}{(2\pi)^3}\int_{T^3}\frac{cosq_1cosq_3dq}{\varepsilon_0(q)-z}$

$$C_1=C_1\lambda(c(z)-e(z))$$

$\Delta_{\lambda}^{(a)}(z) = [1 + \lambda(c(z) - e(z))]^2.$

Demak, $h_{\mu\lambda}^{s}$ va h_{λ}^{a} operatorlarning Fredgolm determinantlari mos ravishda quyidagicha ekan:

$\Delta_{\mu\lambda}^{(s)}(z) \coloneqq \Delta_{\mu 0}(z)\Delta_{0\lambda}(z) - 3\mu\lambda b^2(z),$

bu yerda

$\Delta_{\mu 0}(z) \coloneqq 1 + \mu a(z), \Delta_{0\lambda}(z) \coloneqq 1 + \lambda\big(c(z) + 2e(z)\big)$

$\Delta_{\lambda}^{(a)}(z) \coloneqq [1 + \lambda(c(z) - e(z))]^2.$

lemma isbot bo'ldi.

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