Particle dynamics around a Schwarzschild modified black hole

Husanboy Hoshimov

Doctoral student at Fergana State University

Abstract. In this work, we study the dynamics of test particles around a modified Schwarzschild black hole, taking into account the effects of the Generalized Uncertainty Principle (GUP). Using a metric that incorporates GUP corrections, we derive analytical expressions for the Keplerian (orbital), radial, and vertical epicyclic oscillation frequencies of particles orbiting the black hole. The analysis reveals that as the GUP correction parameter ϵ increases, the Keplerian and vertical oscillation frequencies decrease, whereas the radial oscillation frequency increases. These results indicate that GUP effects have a significant impact on particle trajectories around black holes and could, in principle, be detected through astrophysical observations such as quasi-periodic oscillations (QPOs).

Keywords. Modified gravity, Generalized Uncertainty Principle (GUP), Schwarzschild black hole, particle dynamics, Keplerian frequency, epicyclic frequencies, quasi-periodic oscillations (QPOs).

Introduction and theoretical framework

In this study, we investigate the motion of test particles near a black hole (BH) with corrections from the Generalized Uncertainty Principle (GUP). In Boyer-Lindquist coordinates, the GUP-modified Schwarzschild (S-GUP) metric is given by [1, 2]:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \qquad (1)$$

where the metric function f(r) is expressed as:

$$f(r) = 1 - \frac{2M}{r} + \epsilon \frac{M^2}{r^2}$$

Here, β is the characteristic parameter of the GUP correction. It is important to note that the above metric is not formally derived by solving the field equations. Rather, it has been proposed as a test metric at the lowest order of ϵ to understand GUP effects in Schwarzschild spacetime [1].

Keplerian frequencies

The angular velocity of particles orbiting the BH, as measured by an observer at infinity, is known as the orbital (or Keplerian) frequency, Ω_ϕ, defined as Ω_ϕ=dϕ/dt. For a spherically symmetric spacetime, this definition leads to the following general expression [3,17]:

$$\Omega_\phi = \sqrt{\frac{-\partial_r g_{tt}}{\partial g_{\phi\phi}}} = \sqrt{\frac{f'(r)}{2r}}$$

Specifically, using the metric (1), the final expression takes the following form:

$$\Omega_\phi = \frac{\sqrt{M(r - M\epsilon)}}{r^2}.$$

To express the frequencies in Hertz (Hz), we use the following conversion:

$$v_\phi = \frac{c^3}{2\pi GM} \frac{\sqrt{M(r - M\epsilon)}}{r^2}.$$

Harmonic oscillations

We analyze the small perturbations of a test particle moving in a stable orbit within the equatorial plane of a static BH. The frequencies of radial and vertical oscillations, resulting from small deviations $r \to r + \delta r$ and $\theta \to \pi/2 + \delta\theta$, are defined as:
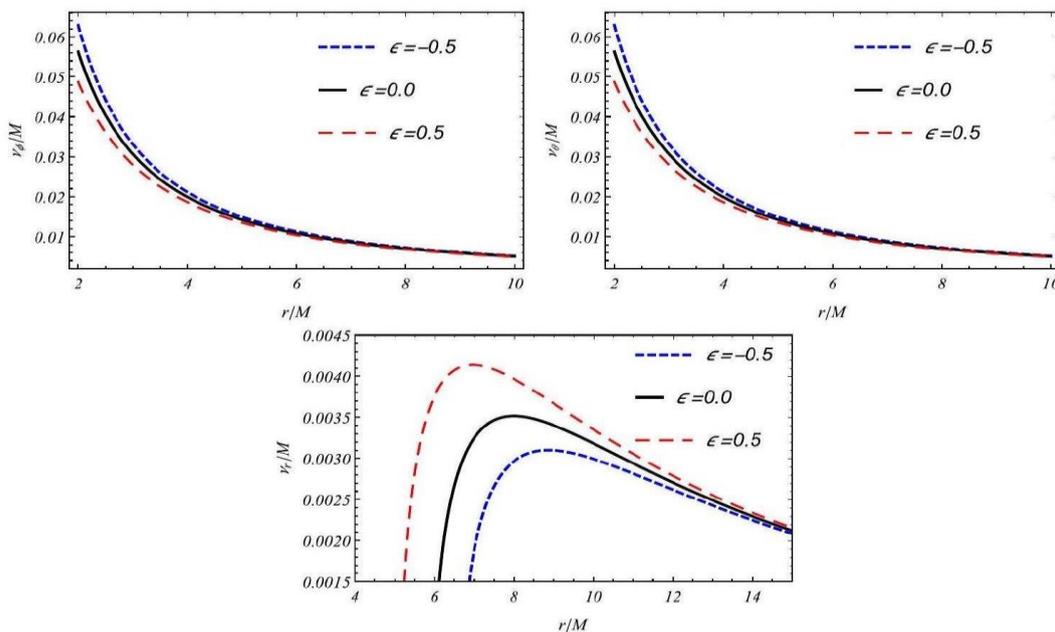

Fig. 1. The figure displays the radial dependence of Keplerian, vertical and radial frequencies for test particles around a S-GUP BH for different values of parameter ϵ.

$$\frac{d^2\delta r}{dt^2} + \Omega_r^2 \delta r = 0, \frac{d^2\delta\theta}{dt^2} + \Omega_\theta^2 \delta\theta = 0$$

Using the equations above, we derive the following expressions for the frequencies in the static BH spacetime:

$$\Omega_r^2 = -\frac{1}{2g_{rr}\dot{t}^2} \partial_r^2 V_{\text{eff}}(r,\theta)\Big|_{\theta=\pi/2} \tag{7}$$

$$\Omega_\theta^2 = -\frac{1}{2g_{\theta\theta}\dot{t}^2} \partial_\theta^2 V_{\text{eff}}(r,\theta)\Big|_{\theta=\pi/2} \tag{8}$$

$$v_r = \frac{c^3}{2\pi GM} \frac{\sqrt{Mr^2(r - 6M) - 4\epsilon^2 M^2 + 9\epsilon M^3 r}}{r^3} \tag{9}$$

$$v_\theta = v_\phi.$$

Analysis of results

The derived expressions allow us to study the influence of the GUP parameter on particle dynamics. Graphical analysis (as depicted, for instance, in Fig. 1) shows that the GUP correction has a negative effect on both the orbital (v_ϕ) and vertical (v_θ) frequencies, meaning these frequencies decrease as the parameter ϵ increases. Conversely, the GUP effect is positive for the radial frequency (Ωr), and the maximum value of the radial frequency increases with an increasing ϵ parameter.

Conclusion

In this study, particle dynamics around a GUP-modified Schwarzschild black hole were examined. Analytical expressions for the Keplerian, radial, and vertical oscillation frequencies were derived, and the impact of the GUP correction parameter ϵ was analyzed. The main conclusions are as follows:

1. The GUP effect reduces the values of the orbital and vertical frequencies.

2. The radial oscillation frequency, in contrast, increases as the GUP parameter grows.

3. These variations alter the properties of stable circular orbits, leading to deviations from the predictions of General Relativity.

These findings could serve as a theoretical basis for testing quantum gravity effects in the future through observations of quasi-periodic oscillations (QPOs) near black holes.

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