Simulation and Modeling of Nonlinear Electromagnetic Effects in Elastic Thin Plates with Complex Shapes

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Abstraction - The paper presents a mathematical model derived from the Hamilton–Ostrogradsky variational principle. By applying the Kirchhoff–Love hypothesis, the original three-dimensional formulation is reduced to a two-dimensional model. A computational algorithm was developed to solve the governing equations, supported by custom software implementation. Numerical experiments were carried out, and the obtained results were analyzed to validate the proposed approach.
Keywords: Mathematical modeling, Hamilton-Ostrogradsky principle, Maxwell's electromagnetic tensor, magneto elastic thin plate of complex configuration.

Introduction

The rapid advancement and widespread use of thin plates and shells with complex geometries, which exhibit nonlinear behavior, emphasize the importance of investigating their deformation under electromagnetic influences. A key research focus is the development of mathematical models, next-generation computational algorithms, and specialized software based on the R-function method to address boundary value problems in magnetoelastic plates and shells of intricate shapes.

Extensive studies on electromagnetic field interaction and conductivity have been carried out by numerous researchers worldwide, including S.A. Ambartsumyan, G.E. Bagdasaryan, M.V. Belubekyan, V.L. Rvachev, L.V. Kurpa, L.V. Molchenko, I.T. Selezov, and M.R. Korotkina. Notable contributions have also been made by academicians V.Q. Qobulov and X.A. Rakhmatulin, as well as professors Sh.A. Nazirov, T. Yuldashev, R. Indiaminov, and F.M. Nuraliev.

Development of a mathematical model of the task.

A mathematical model of the process of nonlinear deformation of a magnetoelastic plate is built on the principle of Hamilton-Ostrogradsky variation. It uses the Kirchhoff-Law hypothesis, the Cauchy relationship, the Lorentz force of Hooke's law, and Maxwell's view of the electromagnetic tensor [1-2]. According to the Kirchhoff-Law hypothesis, there is no deformation of a thin plate along the *Z* coordinate axis, and the displacement projections of the middle plane of the plate are expressed as follows:

$$u_1 = u(x,y,t) - z\frac{\partial w}{\partial x},\ u_2 = v(x,y,t) - z\frac{\partial w}{\partial y},\ u_3 = w(x,y,t), \qquad (2)$$

where: $u, v, w -$ the displacement of the middle plane of the thin plate along the coordinate *(x, y, z)* axes.

Now, the work variations performed by kinetic energy, potential energy, and external forces are reduced to the Hamilton-Ostrogradsky variation principle, resulting in a special derivative equilibrium equation with initial and limit conditions.

$$\begin{cases} -\rho h\frac{\partial^2 u}{\partial t^2}+\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}+N_x+R_x+q_x+T_{zx}=0 \\ -\rho h\frac{\partial^2 v}{\partial t^2}+\frac{\partial N_{yy}}{\partial y}+\frac{\partial N_{xy}}{\partial x}+N_y+R_y+q_y+T_{zy}=0 \\ -\rho h\frac{\partial^2 w}{\partial t^2}+\frac{\partial^2 M_{xx}}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_{yy}}{\partial y^2}+N_{xx}\frac{\partial^2 w}{\partial x^2}+N_{yy}\frac{\partial^2 w}{\partial y^2}+N_{xy}\frac{\partial^2 w}{\partial x\partial y}+ \\ +\left(\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}\right)\frac{\partial w}{\partial x}+\left(\frac{\partial N_{yy}}{\partial y}+\frac{\partial N_{xy}}{\partial x}\right)\frac{\partial w}{\partial y}+N_z+R_z+q_z+T_{zz}=0, \end{cases} \quad (3)$$

Initial and boundary conditions:

$$\begin{cases} \rho h\frac{\partial u}{\partial t}\delta u\Big|_t=0,\ \rho h\frac{\partial v}{\partial t}\delta v\Big|_t=0,\ \rho h\frac{\partial w}{\partial t}\delta w\Big|_t=0, \left(N_{xx}+N_{Px}+N_{Tx}\right)\delta u\Big|_x=0,\ \left(N_{xy}+N_{Py}+N_{Txy}\right)\delta v\Big|_x=0, \\ M_{xx}\delta\frac{\partial w}{\partial x}\Big|_x=0,\ M_{xy}\delta\frac{\partial w}{\partial y}\Big|_x=0,\ \left(N_{yy}+N_{Fy}+N_{Tyy}\right)\delta v\Big|_y=0,\ \left(N_{xy}+N_{Fx}+N_{Tyx}\right)\delta u\Big|_y=0, \\ M_{yy}\delta\frac{\partial w}{\partial y}\Big|_y=0,\ M_{xy}\delta\frac{\partial w}{\partial x}\Big|_y=0, \left[N_{xx}\frac{\partial w}{\partial x}+N_{xy}\frac{\partial w}{\partial y}-\frac{\partial M_{xx}}{\partial x}-\frac{\partial M_{xy}}{\partial y}+N_{Pz}+N_{Txz}\right]\delta w\Big|_x=0, \\ \left[N_{yy}\frac{\partial w}{\partial y}+N_{xy}\frac{\partial w}{\partial x}-\frac{\partial M_{yy}}{\partial y}-\frac{\partial M_{xy}}{\partial x}+N_{Fz}+N_{Tyz}\right]\delta w\Big|_y=0. \end{cases}$$

Computational algorithm for numerical solution of the task.

Algorithm for calculating the geometric nonlinear deformation processes of electromagnetic thin plates [4]:

1. Construction of solution structures corresponding to limit conditions
2. Construction of discrete equations with respect to spatial variables
3. Solving discrete equations and finding unknown components of solution structures.
4. Determining the normal displacements of the middle surface of the plate

To calculate the unknowns in the equation of motion using the given algorithm, the unknown displacement coefficients are determined using a combination of the Bubnov-Galerkin method of variation, the Gaussian square method, the Gaussian method Nyumark and the iteration method.

The limit equation of a magnetoelastic plate of complex configuration is constructed by the R-function [3].

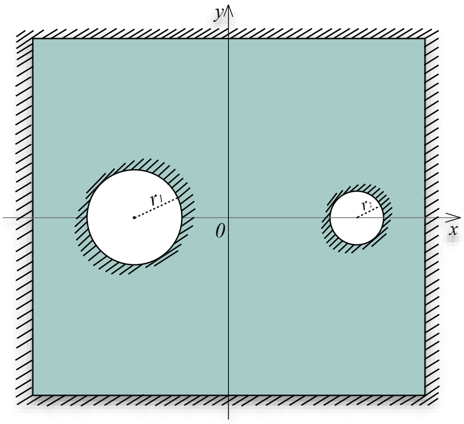

Figure 1. Magnetic plate with complex configuration.

$$f_1 = \frac{(a^2 - x^2)}{2a} \ge 0,\ f_2 = \frac{(b^2 - y^2)}{2b} \ge 0,\ f_3 = \frac{(x^2 + y^2 - r^2)}{2r} \ge 0,\ \omega = (f_1 \wedge f_2) \wedge f_3.$$

where $f_1, f_2, f_3 -$ the functions representing the field. $\omega -$ a normalized function representing the field.

The iteration method is used to find the displacements $u_i(x,y,t), v_i(x,y,t), w_i(x,y,t)$ of the middle surface of the plate Y from the system of equations formed, and a numerical solution is obtained [5].

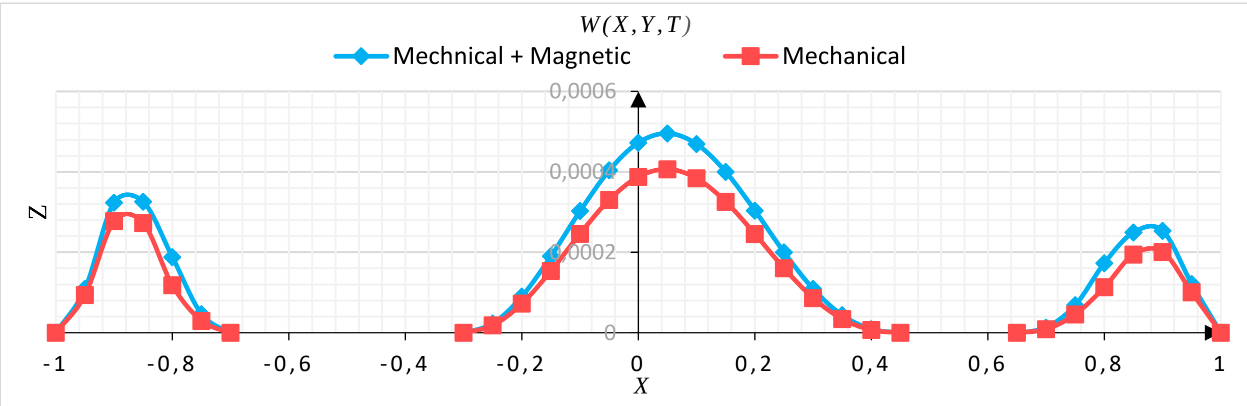

Figure 2. Bending of a magnetoelastic thin plate with an asymmetrical complex configuration.

Conclusion

The study addressed the geometric nonlinear deformation of thin plates with complex configurations under electromagnetic field influences. A novel mathematical model was formulated, and corresponding computational algorithms, together with a software tool, were developed to enable numerical simulations. Symmetric and asymmetric configurations were examined, with results presented in both tabular and graphical forms for comprehensive analysis. The findings provide a methodological basis for further investigations and the solution of related applied problems.

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